but not

$$x \in y \equiv y \in x.$$

Inclusion does not imply a hierarchy, whereas membership does. Instead of moving back and forth freely, as in inclusion, membership constrains us to move in only one way.

We have accumulated three relations which are peculiar to class-inclusion and excluded from membership: inclusion, transitivity, and symmetry. And we already had one property of individuals, namely, their concreteness, which prevents them from being classes.

We may now sum up the difference in identity for individuals and for classes. For individuals it means being the same individual (identity); for classes it means mutual inclusion. Again, conflict for individuals is represented in logic by contradiction for classes, $(\dot{A})+(-A)=0$. Thus the inclusion of classes and the membership of individuals cannot be reduced to one another. They are in the peculiar situation that they are related by dependence and at the same time are independent; without one, the other would not exist, yet they also have differences.

JAMES K. FEIBLEMAN

TULANE UNIVERSITY

BOOK REVIEWS

Decision Making; an Experimental Approach. By DONALD DAVIDSON and PATRICK SUPPES, in collaboration with SIDNEY SIEGEL. Stanford, California: Stanford University Press, 1957. 121 pp. $3.25.

Although individual decision making is studied in economics, philosophy, and psychology, one hardly anticipates anything like a union of the three disciplines in a book entitled *Decision Making.* Yet in this slim volume by two philosophers and a psychologist several axiomatic utility models, similar in conception to those of von Neumann and Morgenstern and of F. P. Ramsey, are developed and two empirical tests of the models are reported that employ methods of experimental psychology. The book is not, as its title suggests, a systematic exposition of a field, but rather a collection of three previously unpublished research papers plus an introduction.

Considering the amount of decision theory produced in the past decade, actual confrontations of theory by data are extremely rare.

The apparently plausible terms employed in most of the models are surprisingly difficult to coördinate with terms designating experimental operations, so, with this in mind, the authors have developed axiom systems that are much more readily and completely testable. However, I suspect that many readers will be disappointed because, on the one hand, their data, while encouraging, are still sufficiently fuzzy to be inconclusive and, on the other hand, the price paid to get what at present are probably the most satisfactory data in this field are theories so special and complex that one can fairly doubt that they will arrest the attention of non-specialists.

The guiding notion here, as in much of decision theory, is the assumption that an individual's choices among actions whose outcomes are contingent upon chance events meet the following restriction known as the expected utility hypothesis: one can construct a numerical subjective evaluation of worth, or utility function, ϕ, over the outcomes and a numerical subjective evaluation of probability, s, over the events in such a way that the action having the largest expected utility relative to his subjective probability is the one he chooses. The problem is twofold: first, using primitives that are more or less easily realizable empirically, to give axiomatizations that justify the above supposition; and, second, to devise practical ways to disentangle these two subjective functions so that they can be measured independently, empirical predictions made, and the theory tested.

For the moment, let the outcome be money. Historically, the first tack assumed ϕ to be linear in money and s to be equal to objective probability; but at least as early as D. Bernoulli it was known that expected monetary values do not give correct predictions. Bernoulli proposed the logarithm for utility, but this was equally *ad hoc* and it was not until von Neumann and Morgenstern's 1947 theory that a successful axiomatization of the expected utility hypothesis was effected. Their model, which continued to be based upon objective probabilities, was subjected to an empirical test by Mosteller and Nogee, but, as the authors point out in detail, this experiment was incomplete and inconclusive. In addition, there has been little encouragement from psychology that people abide by the calculus of probabilities when events have clear-cut objective probabilities. And, more perversely, individuals often seem able to reach decisions when no objective probabilities can be reasonably assigned to the events involved. Indeed, among psychologists it has seemed much more plausible to assume that utility is linear with money and to try to determine subjective probabilities from the data. Not until 1954, when Savage synthesized von Neumann and Morgenstern's utility and de Finetti's subjective probability

notions, was there a theory in which both functions are subjective; however, his axiomatization is hardly testable in the laboratory. Lost in the rush of this research, until emphasized by the authors, was a little known paper of Ramsey's that appeared 27 years ago in his *The Foundations of Mathematics*, which anticipated much of the current work and contained suggestions that have been more fully explored in the present volume.

Consider the simple one-person game,

$$\begin{array}{cc} & \begin{array}{cc} \text{Option 1} & \text{Option 2} \end{array} \\ \begin{array}{c} \mathrm{E} \\ \bar{\mathrm{E}} \end{array} & \left[\begin{array}{cc} x & u \\ y & v \end{array}\right], \end{array}$$

where the person selects a column and a chance event a row, and together they determine the payoff to the person: x if he chooses option 1 and the event E occurs, y if he chooses option 1 and E does not occur, etc. The expected utility comparison, therefore, is $\phi(x)s(\mathrm{E})+\phi(y)s(\bar{\mathrm{E}})$ vs. $\phi(u)s(\mathrm{E})+\phi(v)s(\bar{\mathrm{E}})$. If one can find an event that is subjectively just as likely to occur as not, i.e., an event satisfying $s(\mathrm{E}) = s(\bar{\mathrm{E}})$, then, by cancellation, the comparison reduces to one entirely between utilities, i.e., $\phi(x) + \phi(y)$ vs. $\phi(u) + \phi(v)$. Empirically, the authors hold, such an event is one that, independent of the particular values of x and y, leads to indifference between the options when $u = y$ and $v = x$. Coins and ordinary dice do not have this property! They manufactured several dice with nonsense syllables identifying the faces that do; but, unfortunately, they fail to report the data concerning these events.

Given such an event, then whenever a subject is indifferent between the two options we must have $\phi(x) - \phi(u) = \phi(v) - \phi(y)$, which suggests dealing only with payoffs "equally spaced in utility." An axiomatization is given for preferences among payoffs which are equally spaced that leads to the desired representation theorem. Testing the model is tricky since, obviously, one cannot choose the money payoffs in advance; they must be chosen experimentally for each subject so that they are equally spaced in utility, which demands rapid calculations by the experimenter to "zero in" on the correct values. Actually, because the payoffs were only changed in increments of one cent, these sums could only be determined within upper and lower bounds—bounds that are none too good initially and become progressively worse because the errors accumulate as one determines the utility of new sums in terms of previously estimated utilities.

Of 19 subjects, 15 exhibited behavior that, within the accuracy

of the methods, was completely consistent with all predictions of the model. Predictions made by assuming that a person chooses the option with the larger expected money return were definitely less satisfactory. In 12 of these 15 cases the resulting utility functions were not linear in money (even though the range spanned less than $1). Eight of the subjects were re-run at a later time, and, of these, seven gave substantially the same results. All in all, very gratifying results except for the ambiguities arising from the approximate determination of the payoffs.

The primary limitation of this model is its demand that the payoffs be equally spaced in utility: it requires delicate experimental successive approximations when money payoffs are used and it is probably impossible to apply when the payoffs are non-monetary. Acknowledging this, the authors next turn to procedures that might be suitable when the payoffs are fixed in advance. Clearly, in general the subjects will exhibit choices, not indifferences, and so the relevant equations become inequalities of the form $\phi(x) + \phi(y) \gtrless \phi(u) + \phi(v)$. To solve these, the authors apply some of the methods of linear programming; however, because the inequalities were found to be inconsistent in practice, it was necessary to introduce an additive "threshold of preference" θ to the left side and then to search for solutions with minimum θ.

The experimental payoffs were phonograph records and the subjects, music students. Although it is true that the linear programming model yields somewhat better predictions than either a simple random model or an ordinal model, it is nevertheless not particularly impressive. In addition, the size of the threshold is large in most cases—about the same as the increments between adjacent records. Much of this they attribute to intransitivities in the subjects' preference-patterns among the records.

The fourth and final chapter, which I shall not discuss in any detail, presents a model that allows some pairs of alternatives to be incomparable—a realistic assumption almost never made in the past—and establishes an expected utility type representation theorem. No empirical tests are reported.

Basic to these studies are two commitments, as much philosophical as empirical in nature, that are currently being questioned. The first is the assumption that choices between options can be described as always going one way or the other, in which case they can be represented algebraically, as they are in this book. Possibly subjects do not discriminate perfectly between all options, in which event probabilistic models are required; however, it is reasonably clear that they do discriminate perfectly between some (e.g., if $x > u$ and $y > v$), which means that the model must be a complex

mixture of perfect and imperfect discriminations. But this may be necessary. The second commitment is to the elegantly simple expected utility hypothesis which is so desirable in the rest of decision theory. Some workers are beginning to feel that this may have to be abandoned, at least in any detailed description of decision making. For example, such may be the case if one can find two events having in some sense the same subjective probabilities for a subject, but the estimate of one is based upon an extensive sample and that of the other upon a very small sample. Should his confidence in the estimates affect his decisions, as it appears to, then the expected utility hypothesis is not tenable because at least two numbers are needed to represent his characterization of the events.

In summary, this clearly written book should be of considerable interest to specialists, for whom it is intended, but probably it will not be widely read by those not directly concerned with technical developments in decision theory. In spite of the excellent, but brief, discussion of previous work in the first chapter and the beginning two sections of the second, it is not a general introduction to the area; rather it is a technical report from one of the most stimulating centers for research in individual decision theory in the country.

R. DUNCAN LUCE

HARVARD UNIVERSITY

The Philosophy of Karl Jaspers. Edited by PAUL ARTHUR SCHILPP. New York: Tudor Publishing House, 1957. 918 pp. (The Library of Living Philosophers.) $10.75.

Professor Schilpp has added another excellent volume to his Library of Living Philosophers with *The Philosophy of Karl Jaspers.* This was a good choice. Jaspers, together with Martin Heidegger, represents German philosophy of today which has reached, thanks to these two thinkers, a new peak worthy of Germany's great philosophical tradition.

After having read this imposing book, the reviewer's respect for a difficult job well done is very high. It is a complaint voiced time and again in Anglo-American philosophical circles that German philosophy is hard to understand and almost impossible to translate. This proverbial dictum is disproved by the present volume. The editor, to be sure, has achieved this admirable result only by virtue of a long, patient, and, we suspect, often frustrating devotion to his subject.

The first difficulty posed by any original thinker is his terminology. In Jaspers' philosophy age-old philosophical issues are